[Shapique*, 5(1): Jaanuary, 2016] ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

FUZZY QUEUE WITH ERLANG SERVICE MODEL USING DSW ALGORITHM

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ABSTRACT

This paper intends a method to construct the membership functions using DSW alogorithm of the Performance measures in queueing systems where the inter-arrival and service time are Fuzzified. We intend a Fuzzy nature in FM/FE$_k$ /1 queue model whose arrivals and service rate where E$_k$ denoted erlang probability distribution with k number of phases .The membership functions of the performance measures for the Queueing model FM/FE$_k$ /1 is defined by applying the DSW(Dong, Shah & Wong) algorithm where the arrival and service rate of this model FM/FE$_k$ /1 are Fuzzy numbers.To determine the validity of the proposed method numerical example is illustrated.

KEYWORDS: **Keywords: Queueing theory, Performance measure, α - cut, Membership function, DSW algorithm.**

INTRODUCTION

Queueing models has many application including telecommunication, traffic engineering and computing .All probability queueing models have assumed poison input and exponential service times. In various real-world conventions may be rather limiting, especially the assumption regarding service times being distributed exponentially. The fuzzy queues are much more truthful than the normally used crisp queues. In practice the arrival rate, service rate are commonly defined by linguistic terms such as fast, slow or moderate whcih can be best described by the Fuzzy sets. Clearly when the arrival and service rate are fuzzy the performance measure of the queue also is fuzzy as well. Queuing models in Fuzzy have been described by researchers like Li and Lee [6], Buckely [1], Negi ang Lee [7], Kao et al & Chen [2,3],they have analyzed fuzzy queues using zadeh's extension principle, Zadeh[9]. Recently, Chen [2, 3] developed $(FM\,/\,\mathrm{FM}/1)\ \&\,(FM\,/\,\mathrm{FM}/\,\mathrm{k})$ Based on Zadeh's extension principle analytical results for two fuzzy queueing systems derived by [10] Li and Lee. But the performance measures are not completely described. Srinivasan [9] discussed DSW approach for M/M/ 1 model. This paper intent to follows α cut approach to decompose a fuzzy queue into a family of crisp Queues .when α -varies the DSW algorithm is used to describe the family of crisp queues. The solutions from the DSW algorithm derive the membership functions of the crisp queues.To validate the proposed approach the fuzzy queue $FM\,/\,\mathrm{F}E_k\,/1$ where FM & $\mathrm{F}E_k$ represents fuzzified exponential distribution and fuzzified Erlang distribution respectively.

PROBLEM FORMULATION

Consider a general queueing system in which the customer arrive at a single-server facility with arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$,where $\tilde{\lambda}$ (Poisson rate) is fuzzy in nature and $\tilde{\mu}$ (Erlang service rate) made up of K exponential phases.Let $\mu_{\tilde{\lambda}}(x)$ and $\mu_{\tilde{\mu}}(x)$ are membership function of arrival rate and service rate respectively.

$$\tilde{\lambda} = \left\{x, \mu_{\tilde{\lambda}}(x) / \mathrm{x} \in S(\tilde{\lambda})\right\} \text{ and } \tilde{\mu} = \left\{\mathrm{y}, \mu_{\tilde{\mu}}(\mathrm{y}) / \mathrm{y} \in S(\tilde{\mu})\right\}$$

Where $S(\tilde{\lambda})$ and $S(\tilde{\mu})$ are the supports of $\tilde{\lambda}$ and $\tilde{\mu}$ which denote the universal set of the arrival rate and service rate respectively.

Denote α cuts of $\tilde{\lambda}$ and $\tilde{\mu}$ as

$$\lambda = \left\{ x \in X / \mu_{\tilde{\lambda}}(x) \geq \alpha \right\}$$

$$\mu = \left\{ y \in Y / \mu_{\tilde{\mu}}(\mathrm{y}) \geq \alpha \right\}$$

The inter arrival and service time can be represented by different levels of confidence intervals.

$$\mu_{\rho(\tilde{\lambda},\tilde{\mu})} = \begin{cases} L_S(z), & z_1 \leq z \leq z_2 \\ 1, & z_2 \leq z \leq z_3 \\ L_S(z), & z_3 \leq z \leq z_4 \end{cases}$$

Where $z_1 \leq z_2 \leq z_3 \leq z_4$ and $L(z_1) = R(z_4) = 0$ .An approximate method of extension is propagating fuzziness for continuous valued mapping determined the membership functions for the output variable

2.1 THE $FM / \mathrm{F}E_k / 1$ QUEUE

In this model, unit is served in K phases. A new arrival creates k phases of service and departure of one unit reduces K-phases of service

The expected number of customers in the system

$$L_s = \left(\frac{K+1}{2K}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) + \frac{\lambda}{\mu}$$

Expected number of customer in the queue

$$L_q = \left(\frac{K+1}{2K}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right)$$

Expected time a customer spends in the system

$$W_s = w_q + \frac{1}{\mu}$$

Expected time a customer spends in the queue

$$W_q = \left(\frac{K+1}{2k}\right)\left(\frac{\lambda}{\mu(\mu-\lambda)}\right)$$

INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_1 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$$I_1 = [a,b],\ a < b\ ; I_2 = [\mathrm{c},\mathrm{d}],\ c < d$$

Define a general arithmetic property with the symbol *, where * = [+, -, ×, ÷] symbolically. The operation.

$$I_1 * I_2 = [a,b] * [\mathrm{c},\mathrm{d}]$$

represents another interval. The interval calculation depends on the magnitudes and signs of
the element a, b, c, d.

$$I_1 + I_2 = [a+c, b+d]$$

$$I_1 - I_2 = [a-d, \mathrm{b}-c]$$

$$\mathrm{I}_1 \bullet \mathrm{I}_2 = [\min(\mathrm{ac},\mathrm{ad},\mathrm{bc},\mathrm{bd}), \max(\mathrm{ac},\mathrm{ad},\mathrm{bc},\mathrm{bd})]$$

$$I_1 \div I_2 = [a,b] \bullet \left[\frac{1}{d},\frac{1}{c}\right] \text{ Provided d,c} \neq 0$$

$$\alpha[a,b] = \begin{cases} [\alpha a, \alpha b], \alpha > 0 \\ [\alpha b, \alpha a], \alpha < 0 \end{cases}$$

DSW ALGORITHM

DSW (Dong, Shah and Wong) algorithm is one of the approximate method to make use of intervals at various α -cut levels in defining membership functions.. The DSW algorithm streamlines the manipulation of the extension principle for continuous valued fuzzy variables, such a fuzzy numbers defined on the real line. It avoids the irregularity in the output membership function due to application of the discrimination eaching on the fuzzy variable domain, it can prevent the spreading of resulting functional expression by conventional interval analysis method.

Any membership function which is continuous can be represented by a continuous curve of α-cut N term from α = 0 to α = 1. Suppose we have single input mapping given by $y = f(x)$ that is to be extended for membership function for the selected α cut level.

The DSW algorithm [3] consists of the following steps:

1. Choose the value of the α cut in [0,1]
2. Find the intervals in the input membership functions that corresponds to this α.
3. Using standard binary interval operations, calculate the interval for the output membership function for the selected α- cut level.
4. Continue the steps 1 -3 for various values of α to complete a α- cut representation of the solution.

NUMERICAL EXAMPLE

Consider an integrated a system in which the service consists of two phases .both the arrival and service rate are trapezoidal fuzzy numbers denoted by $\tilde{\lambda} = [3,4,5,6]$ and $\tilde{\mu} = [14,15,16,17]$ per minute respectively. Let's evaluate the performance measure .The confidence interval at α are $[3+\alpha, 6-\alpha]$ and $[14+\alpha, 17-\alpha]$

Consider x=$[3+\alpha, 6-\alpha]$ and y=$[14+\alpha, 17-\alpha]$ with k = 2

$$L_s = \left(\frac{K+1}{2K}\right)\left(\frac{x^2}{\mathrm{y(y}-x)}\right) + \frac{x}{y}$$

$$L_q = \left(\frac{K+1}{2K}\right)\left(\frac{x^2}{\mathrm{y(y}-x)}\right)$$

$$W_s = w_q + \frac{1}{y}$$

$$W_q = \left(\frac{K+1}{2k}\right)\left(\frac{x}{\mathrm{y(y}-x)}\right)$$

Table: The α -cuts of $L_q, W_q, L_s \,\&\, W_s$ at α values

α	L_q	W_q	L_s	W_s
0	[0.0284, 0.2411]	[0.0095, 0.0402]	[0.2048, 0.6696]	[0.0683, 0.1116]
0.1	[0.0309 , 0.2258]	[0.0100, 0.0383]	[0.2143, 0.6442]	[0.0691, 0.1092]
0.2	[0.0336, 0.2115]	[0.0105, 0.0365]	[0.2241, 0.6200]	[0.0700, 0.1069]
0.3	[0.0365, 0.1981]	[0.0111, 0.0348]	[0.2341, 0.5967]	[0.0709, 0.1047]

0.4	[0.0396, 0.1856]	[0.0116, 0.0331]	[0.2444, 0.5745]	[0.0719, 0.1026]
0.5	[0.0428, 0.1739]	[0.0122, 0.0316]	[0.2550, 0.5532]	[0.0728, 0.1006]
0.6	[0.0463, 0.1628]	[0.0129, 0.0302]	[0.2658, 0.5327]	[0.0738, 0.0986]
0.7	[0.0500, 0.1525]	[0.0135, 0.0288]	[0.2770, 0.5130]	[0.0749,0.0968]
0.8	[0.0539, 0.1427]	[0.0142, 0.0274]	[0.2885, 0.4941]	[0.0759, 0.0950]
0.9	[0.0581, 0.1336]	[0.0149, 0.0262]	[0.3003, 0.4759]	[0.0770, 0.0933]
1	[0.0625, 0.1250]	[0.0156, 0.0250]	[0.3125, 0.4583]	[0.0781, 0.0917]

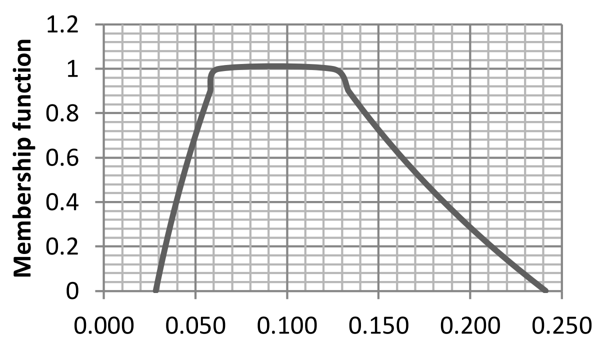
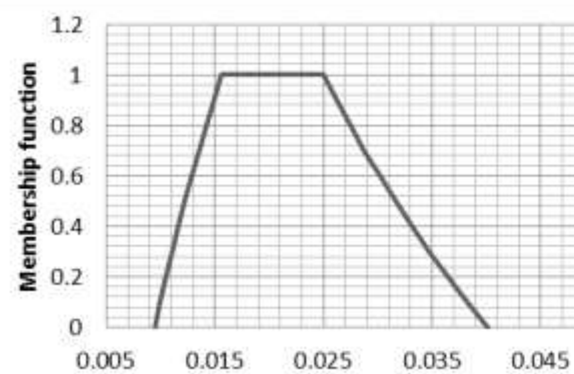


Fig : 1 membership function of the expected length Lq



Membership function of waiting time Wq

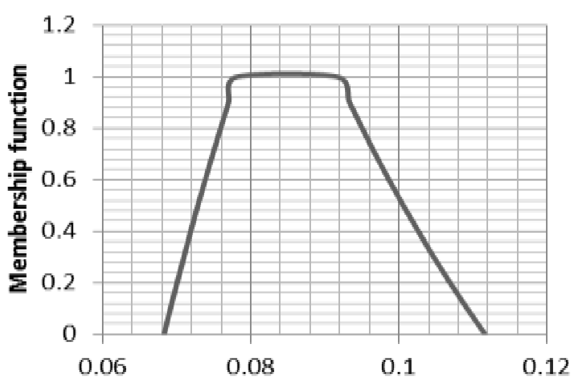


Fig : 3Membership function of the waiting time Ws

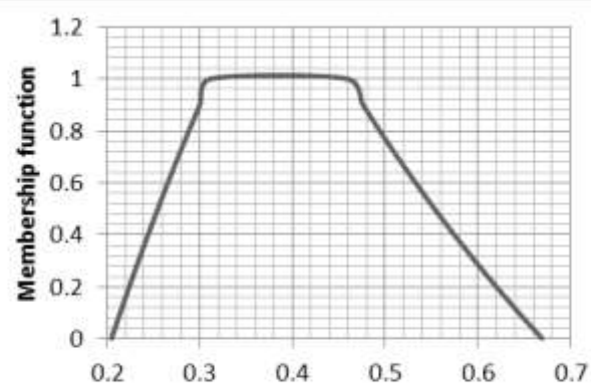


Fig:4 membership function of the queue length in system Ls

With the help of MATLAB software we perform α – cuts of arrival , service rate and fuzzy expected number of jobs in queue at eleven distinct levels of α 0, 0.1, 0.2, 0.3, …1. Crisp intervals for fuzzy expected number of jobs in queue at different possibility α levels are presented in above Table. Similarly the performance measure such as Ls Lq, Wq and Ws also derived in Table. The – cut represents the possibility that these four performance measure will lie in the associated range. Specially, α = 0 the range, the performance measures could appear and for α = 1 the range, the performance measures are likely to be. For example, while these four performance measures are fuzzy, the most likely value of the expected queue length *Lq* falls between 0.0625 and 0.1250 and its value is impossible to fall outside the range of 0.0284 and 0.2411 similarly the expected length of the system falls between 0.3125 and 0.4583 and won't fall outside the range of [0.2048, 0.6696].The above data will be very suitable for designing a queueing system.

CONCLUSION

The concept of fuzzy has been applied to queuing systems to provide broader application in many areas. This paper develops the inter-arrival and service time are fuzzified , according DSW algorithm, the performance measures such as the expected number customer in the system , expected number customer in the queue, expected waiting time in the system and queue will be fuzzy. Numerical example shows the efficiency of the algorithm

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